Senior problems

S187. Find all positive integers n for which the interval

$$\left(\frac{1+\sqrt{5+4\sqrt{24n-23}}}{2}, \frac{1+\sqrt{5+4\sqrt{24n+25}}}{2}\right)$$

contains at least one integer.

Proposed by Titu Andreescu, University of Texas at Dallas, USA

S188. Let $a \geq b \geq c$ be the side-lengths of a triangle ABC in which $b+c \geq 2a\cos\frac{\pi}{5}$. Denote by O and I the circumcenter and the incenter of this triangle, respectively. Prove that circle centered at O and having radius OI lies entirely inside triangle ABC.

Proposed by Ivan Borsenco, Massachusetts Institute of Technology, USA

S189. Let a, b, c be real numbers such that $a < 3$ and all zeros of the polynomial $p(x) = x^3 + ax^2 + bx + c$ are negative real numbers. Prove that $b + c \neq 4$.

Proposed by Titu Andreescu, University of Texas at Dallas, USA

S190. Let a, b, c be positive real numbers. Prove that

$$\frac{1}{2a^2+bc} + \frac{1}{2b^2+ca} + \frac{1}{2c^2+ab} \leq \frac{1}{9}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)^2.$$

Proposed by Arkady Alt, San Jose, California, USA

S191. Prove that for any positive integer k the sequence $(\tau(k+n^2))_{n\geq 1}$ is unbounded, where $\tau(m)$ denotes the number of divisors of m.

Proposed by Al-Yazeed Ibrahim Basyoni, Saudi Arabia

S192. Let s, R, r and r_a, r_b, r_c be the semiperimeter, circumradius, inradius, and exradii of a triangle ABC. Prove that

$$s\sqrt{\frac{2}{R}} \leq \sqrt{r_a} + \sqrt{r_b} + \sqrt{r_c} \leq \frac{s}{\sqrt{r}}.$$

Proposed by Arkady Alt, San Jose, California, USA